# An Curved-folding Algorithm to Adapt to The Freeform Surface: Design and Fabrication

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## 1. Introduction

In the field of architecture ,design and art, curved-folding has the exciting potential of a particular technique. It can provide architects with the generation of complex geometries that designing spatial and performative capacities. Most of the researches have used curved-folded tessellations that applied to a predefined global geometry with a top-down method.[1)2)3)](Figure 1.) However, the pavilions are made of numerous self-similarity of sheet material, They used a method of discrete differential geometry to produce developable strips of triangles. The triangles producing fabrication data that ensures consistent assembly. Consequently, the constructing process is demanding due to complex self-similarity of triangle. The challenge of curved-folding is to understand and measure the three-dimensional configuration of curved folded geometry.

Figure 1.Applications of curved folding in architecture.
Epps and Verma,2013(left); Bhooshan et al.2015(middle); Chandra et al.,2015(right)

The objective of this research is to developing an curved-folding technique of design and fabrication. The technique can preserve the continuity of developable surface with curved-folded sheet fitting on free-form surface. In this study, we generated and unfolded curved-folding paper in the digital environment by using Kangaroo, a plugin of Grasshopper with physical engine, has to balance the effects of multiple rules, through Rhinoceros environment. The essential contribution of this paper is a proposed design and fabrication process for architects and designers. They can easily used in not only early stage of design thinking but also fabrication.

The article presents the detail of Grasshopper settings for curved folding simulation by explaining how the proposed algorithmic framework and the parameter of Grasshopper components. Lastly, it discusses the shortcomings and the potentials of the design and fabrication process.

## 2.Geometry of paper :Developable surface

Developable surface have been researched widely in mathematics and computer graphics fields. The early mathematical researches attempted to understand and describe the behavior of the paper sheet along the curves. The sheet paper is one of specific class of surface in mathematics, which is known as a developable surface. [4)]

A developable surface is curved surface and is isometrically transformable, capable of changing to a plane without compressing or expanding. A developable surface consists of numerous line elements, which means that it is ruled surface. In mathematical field, the developable surface is expressed by Equation

$$X(s,t) = p(s) + t \cdot \frac{p''(s) \times p'''(s)}{|p''(s)|^2} \tag{1}$$

It should be point out that the pavilion we studied describe three-dimensional geometry of developable surface based on a priori knowledge of developable surfaces such as “the sum of the angle between edges around a point must be 360 degree.” [5)]

## 3. Algorithm and Design process

The algorithms developed whitn the Grasshopper are visual programming environment and Using the plugin, Kangaroo, to set mathematical rules of developable surface. In order to generate the mesh of developable surfaces, we converted the mathematical rule into physical method and existing components

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of kangaroo through defining the combination of components in Grasshopper.

### 3.1 DESIGN CURVED-CREASES PATTERN

We designed curved-crease pattern on the freeform surface in Rhinoceros environment. By operating the command in Rhino,"FlowalongSurf", the curved-crease pattern can easily to adapt to the freeform surface. Also, the curved-crease pattern must be smooth appearance. In other words, the degree of NURBS must be more than 2.

### 3.2 GENERATING MESH OF DEVELOPABLE SURFACES

In order to generate developable surfaces between curved-creases pattern, the numerous effects of forces in Kangaroo (Figure 2.) must be balanced. The forces,"Planarize","OnCurve","EqualLength" and "Anchor" calculate and generate developable surfaces between curves in the meantime.

In this part, the most vital force is "Planarize" because it makes the line elements between each curves describe the developable surface. Furthermore, "OnCurve" comfirms the line elements closely attaches to the curved-crease pattern."EqualLength" try to verify that length of each pair of adjacent segments along the curves equal, and to avoid it degenerating into triangles."Anchor" keep end points of each curves moveless, so the simulation of meshes can describe developable surface completely between curved-crease pattern.

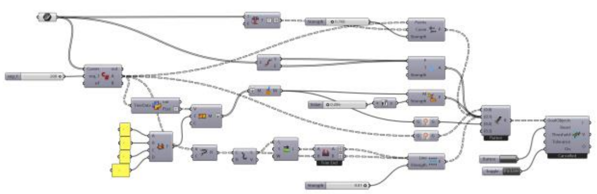

Figure 2. For generating developable surfaces, the proposed definition of Grasshopper to simulate using Kangaroo.

It should be noted that the difficulty in balancing the effect of forces on the Planarize system occurs because we converted the mathematical principles of developable surface into physical method. Namely, It is different from precise mathematical model. Therefore, we didn't run the simulation to convergences , instead, simulation for 50-100 iterations. Figure 3. shows that variation of the line elements during the simulation. The iterations need to be under control due to generating developable surfaces better. During the iteration of kangaroo, checking the sum of neighbouring sector angles is approximate to $2\pi$ in order to redurce the inaccuracy of unfolding the mesh to be flat.

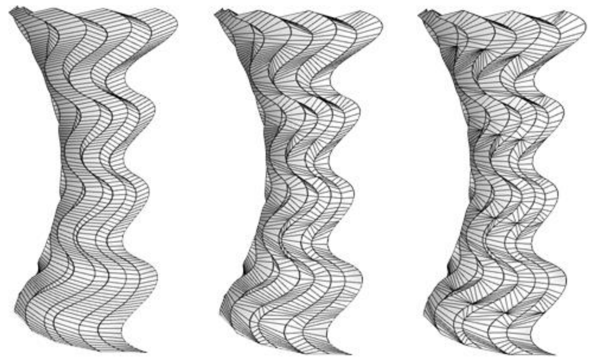

Figure 3. Simulating the developable surfaces between curved-creases from 10 iterations (left) ,50 iterations (middle) and 90 iterations (right).

The main goal of this simulation is to generate the geometries which are relatively similar to developable surfaces as same as possible. Based on the results of testing combinations of force strength values, it can be concluded that there must be one governing force. For example, if the strength of "OnCurve" given to the mesh is high, the strength given to the "Planarize" should be relatively low, and then we start with planarization strength low, and gently slide it up to simulate the developable surfaces. If the strength of "planarize" start high, the line elements can't smoothly move between curved-creases pattern to form the meshes of developable surface.

### 3.3.UNFOLDING THE MESH OF DEVELOPABLE SURFACES

A key difference between our method and the precedents we studied is that we unfold the constructive geometry of mesh instead of unfolding the underlying triangulated mesh. It help fabricate the developable surface and construct easily.

In order to unfold the meshes of developable surfaces between curved-creases pattern, the numerous effects of forces in Kangaroo(Figure 4.) must be balanced. The forces "OnPlane", "Length", "Hinge" and "OnCurve" calculate and unfold the developable surfaces in the meantime.

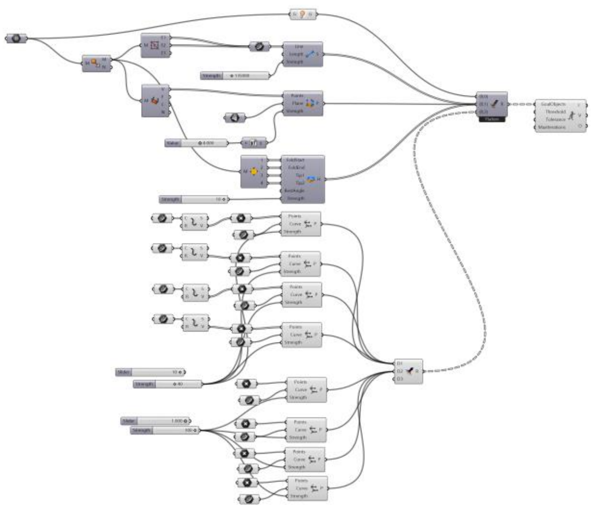

Figure 4. For unfolding the mesh of developable surface, the proposed definition of Grasshopper to simulate using Kangaroo.

In this part, the most crucial is “OnPlane”for the reason that it can force the each vertex coplanar. Inaddition, “Length” try to keep the each edge length of mesh same. Further,“Hinge” unfold the angle between each triangulated mesh to π. In other word, it can relax the folded shape to be flat. It is important to mention that because each quadrilateral mesh divide into triangular mesh, so ”Hinge” works successfully. Finally,”OnCurve”pull every vertex of boundary of folded shape outward.

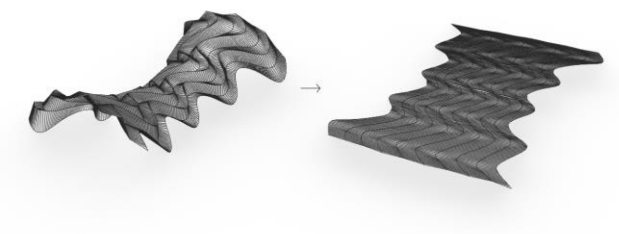

Figure 5. Unfolding the curved-folding paper to be flat

It should be noted that using “zombie” , one of Kangaroo solver, to simulating due to conveniently unfolding the meshes to be completely flat.(Figure 5.) In addition, the strength of “Length” should be relatively high in order to keep the mesh of developable surfaces same. However, balancing the effect of forces have difficulty keeping every edge changeless during the simulation. Importantly, the geometry simulated by this method is a visual approximation and not precise curved folded geometry. Nonetheless, it enable unfolded two-dimensional shapes can be folded and fit on the freeform surfaces.

**4. Paper Model Experiment**

The final stage is to realize the result, we test on a mathematical surface,hyperbolic paraboloid,and then use a sheet paper with curved-creases pattern we designed and folded them manually to testify the feasibility of the algorithm.
After folding paper along the mountain and valley creases, the paper can automatically form the expected shape(Figure 6.).

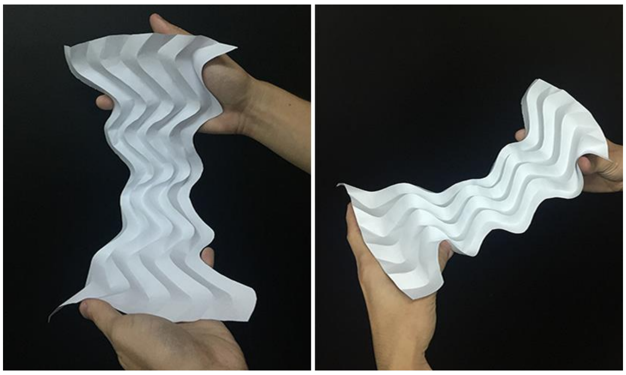

Figure 6. Paper Model Experiment

**5.Result of Simulation on Freeform Surface**

The positive and negative Gaussian curvature can coexist on freeform surface due to the mathematical rule. Figure 8. shows an example of this testing. We simulate the developable surface between curved-crease that we designed on the freeform surface(Figure 7.).

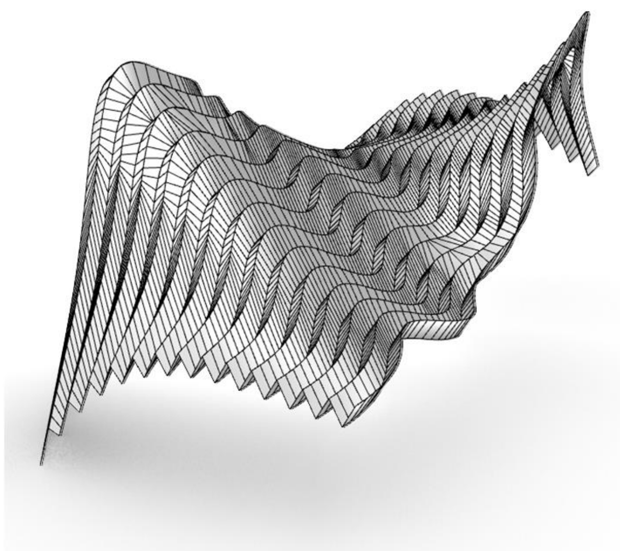

Figure 7. Implementation on the freeform surface.

## 6.Results and Discussion

In our design process of approximating geometry as mentioned in 3.2, we maked a fundamental assumption that inaccuracy of geometry can be tolerated due to using Kangaroo engine to unfold the geometry.

In the design process of generating developable surfaces, it should be pointed out that increasing line elements facilitate better generating mesh of developable surfaces. Further, the sum of the angle between edges around a point have difficulty summing up to 2π (Figure 8.) However, In the design process of unfolding the meshes, the inaccuracy angle of developable surface will be eliminated due to edge of meshes variation(Figure 9.). Namely, we set the strength of "Length" extremely high in order to keep every edge of mesh same, but it turn out to be only approximate same for the reason that unfolding the meshes to be flat is more critical. Finally, the folded model only approximates the visual appearance of the object, which is critical to a designer.

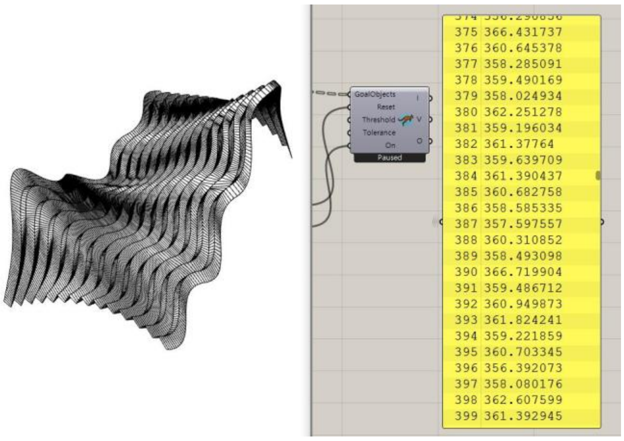


Figure 8. Simulation of generating mesh of developable surfaces (left), the yellow panel(right) shows that sum of the angles between edges around a point are only closely 2π.

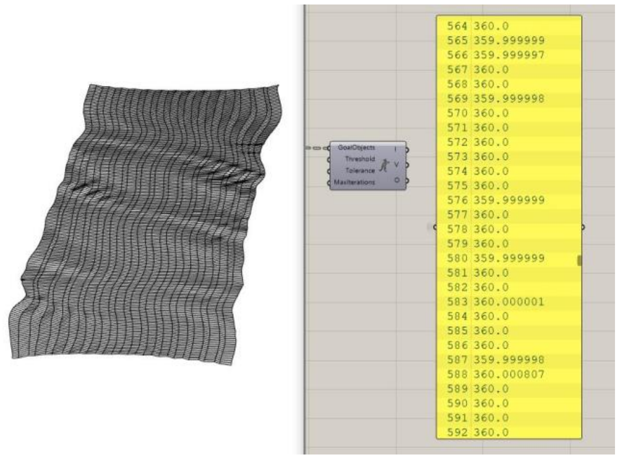


Figure 9. Simulation of unfolding the mesh of developable surfaces (left), the yellow panel(right) shows that sum of the angle between edges around a point are 2π.

## 7.Conclusion

In this research, we demonstrated the inverse technique to design curved-folding paper and a design process to achieve to adapt to the free form surface. Further, the approach presented the fabrication of constructive developable surfaces and is applicable to any freeform surface.

The domain of curved-folding has a strong potential for architecture and manufacture due to aesthetic and flexibility of structure. In recent decades, the development of computer graphics and software of Curved-folding tool facilitate architects and designers with multiple constraints and advance algorithms. The software of designing curved-folding have specific function for design requirements. However, Grasshopper platform provide friendly design environment for architects and designers.

The method presented offer various chances to inform parameters such as line elements of developable surface. Also, it successfully works for intergrating with generating of developable surface and fabrication. We hope to further this approach by enriching it with relevant conmtemporary studies in related origami amd architecture domain. Bhooshan et al.[2)] presented a method applicable to any freeform surface that can be tessellated into convex polygons with curved-folding. Our method has high potential of curved-folding tessellation to adapt to the freeform surface.

**References (sample)**

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